KALMAN FILTERS FOR DYNAMIC POSITION CONTROL OF LARGE SCALE SYSTEMS

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ABSTRACT: The paper derives the model of a large scale system under position control. Its Kalman filter model and the Kalman gain matrix are derived and integrated in the controller. Problems are discussed.

KEYWORDS: Kalman filter, frequency variations, errors, position control, directivity, drift, state estimate, state feedback.

I. INTRODUCTION: In position control noise signals are removed by Kalman filters integrated into control [2,3] system design. The dynamic positioning of large systems is modeled and the Kalman filter effect in control is presented.

Dynamic positioning systems are coupled to other mechanical structures and errors in positioning the primary element, are transfered in this mechanical chain, enhanced, with catastrophic results. Such systems are machine tools, meteorological ballons, platforms, robot arms,etc. Noise induces motion of a freely positioned body,in 6 degrees of freedom. In the case of a body we specify the following objectives:

a. allowable radial position errors $< 3\ \%$ and
b. frequency variations < 0.3 rad/sec

Any low frequency noise induced on the body position, results in appropriate control actions properly activated by the control system, while high frequency noise might lead to actuators failure and unnecessary energy dissipation.

Noise is induced by the following system functions and hardware:

a. position measurements
b. external reasons
c. controller

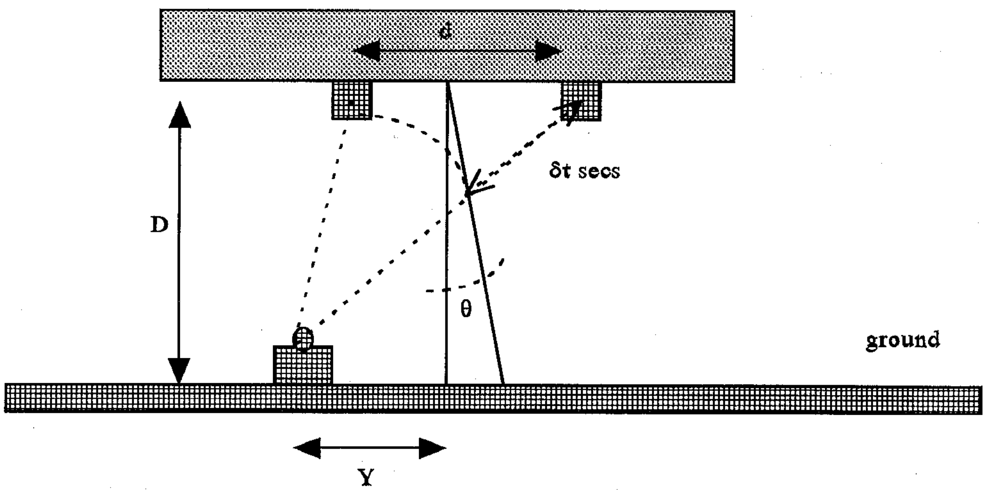

Fig. 1. Large scale system architecture for position control

Position in large scale objects is measured by beacons (based on time-of-flight measurements of ultrasound pulses); any displacement Y caused by noise is:

v is the sound velocity

$$Y = D.\tan\left(\arcsin\frac{v.\delta t}{d}\right) \cong \frac{D.v.\delta t}{d} \text{ where:}$$

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d the beacons separation

D the reference wall

Angle measurement (θ) requires compensation for body roll and pitch. Acoustic noise variance is typically 0.1 m^2 .

For simplification we consider that:

a. only the motions in the sway and yaw direcions, since surge motions are normally decoupled from the sway and yaw motions,

b. the low and high frequency structure motions are determined separately (for simplicity and accuracy reasons) and

c. the total motion is the sum of the above analyzed motions

Induced noise in the system could have the following statistical characteristics and source origin:

(a) highly directive

(b) with an average directivity $\tilde{n}$ and a zero mean component modeled by a random variable $\tilde{\omega}$ with a Gaussian distribution; in case of drift the $\tilde{\omega}$ component has no zero mean value

(c) relatively steady second order forces (main disturbance)

(d) mass inertia and viscous drag and other forces; they are function of the system states and included in the non-linear low frequency model of the under position control system dynamics; the non linear equations can be linearized around an operating point determined by $\tilde{n}$

Forces of nature (a), (b) and (c) are combined and named F_a, and they are proportional to the control signal $\tilde{u}$ of the linearized model.

For a system with a state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & a_{13} & 0 \\ 1 & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_a \\ T_a \end{bmatrix}$$

the elements a_{13} and a_{31} denote the sway to yaw motions interactions. The elements a_{ij} depend on the noise and mean valued disturbances. The states are:

x_2 = the sway position

x_4 = the heading angle

$\dot{x}_2 = x_1 =$ the sway velocity

$\dot{x}_4 = x_3 =$ the heading velocity

For a system model, where the position control actions are amplified to forces states x_5 and x_6 :

$$x_5 = \int [b_1 . (x_5 + u_1)] . dt$$

$$x_6 = \int [b_2 . (x_6 + u_2)] . dt$$

we get

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} -b_1 & 0 \\ 0 & -b_2 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

while the components of force and torque in the sway and yaw directions are:

$$\begin{bmatrix} F_{sway} \\ T_{yaw} \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} F_{sway} \\ T_{yaw} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

The total applied force in the sway direction, F_a, is:

$$F_a = (\gamma_1 . x_5 + \gamma_2 . x_6) + \omega_1 + (e_1 . n_1 + e_2 . n_2)$$

The total applied torque in the sway direction, T_a, is:

$$T_a = (\gamma_3 . x_5 + \gamma_4 . x_6) + \omega_2 + (e_3 . n_1 + e_4 . n_2)$$

The low frequency state space model becomes:

$$\dot{\tilde{x}}_l = \tilde{A}_l . \tilde{x}_l + \tilde{B}_l . \tilde{u}_l + \tilde{D}_l . \tilde{\omega}_l + \tilde{E}_l . \tilde{n}_l$$

where:

$$\underset{\sim}{A_l} = \begin{bmatrix} a_{11} & 0 & a_{13} & 0 & b_1.\gamma_1 & b_1.\gamma_2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 & b_2.\gamma_3 & b_2.\gamma_4 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -b_2 \end{bmatrix},$$

$$\underset{\sim}{B_l} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ 0 & b_2 \end{bmatrix}, \quad \underset{\sim}{D_l} = \begin{bmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \underset{\sim}{D_l} = \begin{bmatrix} b_1.e_1 & b_1.e_2 \\ 0 & 0 \\ b_2.e_3 & b_2.e_4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The low frequency component of the system position presents the following output equation:

$$\underset{\sim}{y_l} = \begin{bmatrix} y_{l_{sway}} \\ y_{l_{yaw}} \end{bmatrix} = \underset{\sim}{C_l}.\underset{\sim}{x_l} \quad \text{where} \quad \underset{\sim}{C_l} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

II. HIGH FREQUENCY POSITION CONTROL:

The noise spectrum is derived by source and system signal analysis or it is known (wind, seawaves, vibrations, etc spectra); for sea:

$$S(\omega) = \frac{a}{\omega^5}.e^{-\frac{b}{\omega^4}} \qquad [m^2.\sec]$$

where:

ω *[rad/sec]* is the frequency

a = *4.894*

b = *3.109 / ($h_{1/3}$)^2*

$h_{1/3}$ [m] = the significant wave height

The worst case spectrum is white noise input and the state equations are:

$$\underset{\sim}{\dot{x}_h} = \underset{\sim}{A_h}.\underset{\sim}{x_h} + \underset{\sim}{D_h}.\underset{\sim}{\omega_h}$$

where:

$$\underset{\sim}{A_h} = \begin{bmatrix} \underset{\sim}{A_h^s} & 0 \\ 0 & \underset{\sim}{A_h^y} \end{bmatrix}, \quad \underset{\sim}{D_h} = \begin{bmatrix} \underset{\sim}{D_h^s} & 0 \\ 0 & \underset{\sim}{D_h^y} \end{bmatrix}$$

$$A_s^h = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4^s & -a_3^s & -a_2^s & -a_1^s \end{bmatrix}, \quad D_s^h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K^s \end{bmatrix}$$

The high frequency component of the position of the vessel is given by the output equation:

$$\underset{\sim}{y_h} = \begin{bmatrix} y_{h_1} \\ y_{h_2} \end{bmatrix} = \underset{\sim}{C_h}.\underset{\sim}{x_h}, \quad \underset{\sim}{C_h} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

By extended Kalman filtering we can get a better estimate [4,5,6].

III. LINEARIZED MODEL:

Let's consider a system with state equation for the low and high frequency:

$$\begin{bmatrix}\dot{x}_l \\ \dot{x}_h\end{bmatrix} = \begin{bmatrix}\underset{\sim}{A_l} & 0 \\ 0 & \underset{\sim}{A_h}\end{bmatrix}\begin{bmatrix}x_l \\ x_h\end{bmatrix} + \begin{bmatrix}\underset{\sim}{B_l} & \underset{\sim}{E_l} \\ 0 & 0\end{bmatrix}\begin{bmatrix}\underset{\sim}{u_l} \\ \underset{\sim}{n_l}\end{bmatrix} + \begin{bmatrix}\underset{\sim}{D_l} & 0 \\ 0 & \underset{\sim}{D_h}\end{bmatrix}\begin{bmatrix}\underset{\sim}{\omega_l} \\ \underset{\sim}{\omega_h}\end{bmatrix}$$

The final position is the sum of the low and high frequency motions, i.e:

$$\underset{\sim}{y} = \underset{\sim}{y_l} + \underset{\sim}{y_h}$$

The position measurement $\underset{\sim}{z}$ is:

$$\begin{bmatrix}z_1 \\ z_2\end{bmatrix} = \begin{bmatrix}\underset{\sim}{C_l} & \underset{\sim}{C_h}\end{bmatrix}\begin{bmatrix}x_l \\ x_h\end{bmatrix} + \underset{\sim}{v} = \underset{\sim}{y_l} + \underset{\sim}{y_h} + \underset{\sim}{v} \qquad \text{where} \quad \underset{\sim}{v} = \begin{bmatrix}v_1 \\ v_2\end{bmatrix}$$

is the white noise random position control effects component.

For known noise state disturbances in the position control system, the system matrices are considered to be constant,while in general they are time varying. The state space equations, in brief, are:

$$\underset{\sim}{\dot{x}} = \underset{\sim}{A}.\underset{\sim}{x} + \underset{\sim}{B}.\underset{\sim}{u} + \underset{\sim}{D}.\underset{\sim}{\omega}$$

$$\underset{\sim}{z} = \underset{\sim}{C}.\underset{\sim}{x} + \underset{\sim}{v}$$

where:

$\underset{\sim}{u}$ includes the deterministic control and measurable disturbance inputs

$\underset{\sim}{\omega}$ represents white process noise input

$\underset{\sim}{v}$ represents the white measurement noise signal

IV. ESTIMATION: For control puprposes it is not the $\underset{\sim}{y} = \underset{\sim}{y_l} + \underset{\sim}{y_h}$ system position that it should be estimated but the $\underset{\sim}{y_l}$ low frequency position component. The question then is to produce an estimate of the $\underset{\sim}{y_l}$ i.e. a $\underset{\sim}{\hat{y}_l}$. By using low frequency components estimated state feedback we build the position controller. By including low and high frequency components we get:

$$\underset{\sim}{\dot{x}} = \underset{\sim}{A}.\underset{\sim}{x} - \underset{\sim}{K}.(\underset{\sim}{C}.\underset{\sim}{x} - \underset{\sim}{z}) + \underset{\sim}{B}.\underset{\sim}{u} = \underset{\sim}{A}.\underset{\sim}{x} - \begin{bmatrix}\underset{\sim}{K_l}(t) \\ \underset{\sim}{K_h}(t)\end{bmatrix}.(\underset{\sim}{C}.\underset{\sim}{x} - \underset{\sim}{z}) + \underset{\sim}{B}.\underset{\sim}{u}$$

$$\underset{\sim}{\hat{y}} = \underset{\sim}{C}.\underset{\sim}{\hat{x}}$$

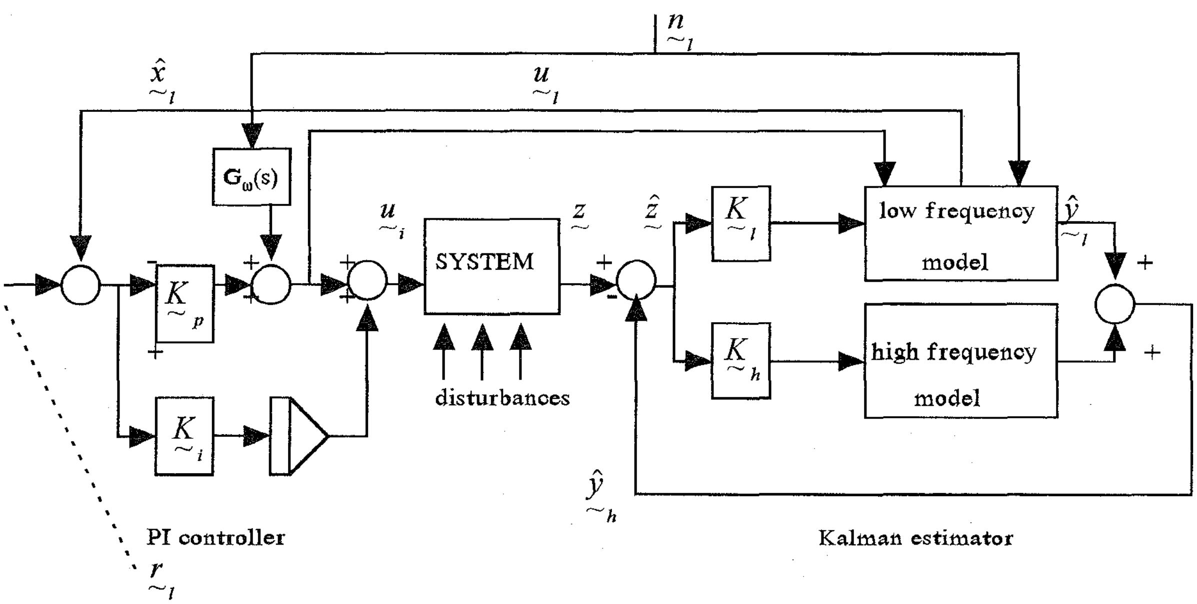

Fig. 2. Dynamic Positioning Control System and Estimator

For given noise covariance the Kalman gain matrix $\underset{\sim}{K}(t)$ is calculated; the measurement covariance matrix is well defined, while the process covariance matrix is not well defined. Integral controller term cancels out effects from unknown disturbances, without affecting the Kalman filter. The controller gain matrices should satisfy classical design and optimal control criteria [1]. Feed-forward control can balance fast disturbances [8].

The number of states in the Kalman filter increases complexity in design, control and calculation; approximations (measurements time lags, nonlinearities, uncertainties, e.t.c.) reduce this complexity and introduce errors in control precision and states estimation. Position control actuators are non-linear. Some low frequency disturbances are treated as unmodelled phenomena.

Cascaded resonant band-rejection filters are used to attenuate with the minimum of phase shift at the lower control frequencies. The Wiener filter is equivalent to the constant gain Kalman filter, but expressed in transfer function form; for non-stationary noise Wiener filter's initial response is suboptimal [1].

V. CONCLUSIONS:

Dynamic positioning of a large is obtained by Kalman filter; the state estimation problem divides naturally into high and low frequency parts. The method's main disadvantage is the need for greater computing power [7], a demand that is overcome thanks to computer development. Instability must allways be detected in time, since position control divergence is possible [9].

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APPENDIX I: EQUATIONS of MOTION: The dynamical equations of a large scale system are:

$$1.044.\dot{u} - r.v = F_{a1} + 0.092.v^2 - 0.138.u.U$$

$$1.84.\dot{v} + r.u = F_{a2} - 2.58.v.U - 1.8.\frac{v^3}{U} + 0..065.r.|r|$$

$$0.2861.\dot{r} = T_a - 0.764.u.v + 0.258.v.U - 0.154.r.|r|$$

where u, v, r are te velocioties of linear disturbances in the surge, sway and yaw directions, U the system velocity relatively to ground i.e. $U = \sqrt{u^2 + v^2}$.

APPENDIX II. The KALMAN GAIN MATRIX: Discrete system and Kalman filter are:

$$x(k+1) = \Phi(k+1,k).x(k) + \Psi u(k) + \Gamma \omega(k)$$

$$z(k) = C x(k) + v(k)$$

$$E\{\omega(k)\} = 0 \qquad E\{\omega(k).\omega^T(m)\} = Q\delta_{km}$$

$$E\{v(k)\} = 0 \qquad E\{v(k).v^T(m)\} = R\delta_{km}$$

where δ_{km} is the Kronecker delta function.

$$\Psi = \int_0^{\tau_1} \Phi(\tau).B.d\tau$$

$$\Gamma = \int_0^{\tau_1} \Phi(\tau).D.d\tau$$

$$\Phi(k+1,k) \overset{\Delta}{=} \Phi(\tau_1)$$

where τ_1 is the sampling interval; the state estimates are:

$$\hat{x}(k+1|k) = \Phi(k+1|k).\hat{x}(k|k)$$

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1).\left(y(k+1) - C.\hat{x}(k+1|k)\right)$$

while:

$$P(k+1|k) = \Phi(k+1|k).P(k|k).\Phi^T(k+1|k) + \Gamma.Q.\Gamma^T$$

The Kalman gain matrix is:

$$K(k+1|k) = P(k+1|k).C^T.\left[C.P(k+1|k).C^T + R\right]^{-1}$$

The error covariance matrix is:

$$P(k+1|k+1) = (I - K(k+1).C).P(k+1|k).(I - K(k+1).C)^T + K(k+1).R.K^T(k+1)$$

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